Université Libanaise
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الجامعة اللبنانية
كلية الاعلام
الفرع الاول

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Exercise 1

Consider the following linear system depending on the real parameter m:

$$\begin{cases} x+y+(1-m)z=m+2 \\ (1+m)x-y+2z=0 \\ 2x-my+3z=m+2 \end{cases}$$

1. Write the system in matrix form.
2. Compute the determinant of the coefficient matrix as a function of m.
3. Discuss, according to the values of m, the number of solutions of the system.
4. Solve the system for each possible case.

Exercise 2

1. Consider the matrix

$$A=\begin{pmatrix} -2 & 0 & 1 \\ -5 & 3 & a \\ 4 & -2 & -1 \end{pmatrix},$$

where a is a real parameter.

1. Compute the trace and the determinant of AAA in terms of a.
2. Knowing that the eigenvalues of A are 0, 3, and −3, determine all values of a.
3. Verify that, for the value(s) of a found, the characteristic polynomial of A is

Exercise 3

Let

$$A = \begin{pmatrix} 2 & 0 & -2 \\ 0 & 4 & 0 \\ -2 & 0 & 5 \end{pmatrix}.$$

1. Find the **eigenvalues** of A.
2. For each eigenvalue, determine a basis of the corresponding **eigenspace**.
3. Decide whether the matrix A is **diagonalizable**. Justify your answer.
4. If A is diagonalizable, compute A^9

Exercise 4

Let $V = M_2(\mathbb{R})$, the vector space of all 2×2 real matrices. Consider the subset

$$W = \{A \in M_2(\mathbb{R}) \mid A + A^T \text{ is a diagonal matrix}\}.$$

a) Show that W can be written in the form

$$W = \left\{ \begin{pmatrix} a & b \\ -b & d \end{pmatrix} \middle| \, a, b, d \in \mathbb{R} \right\}.$$

b) Determine whether W is a subspace of V. Justify your answer by checking the subspace criteria (zero vector, closure under addition, closure under scalar multiplication).

Exercise 5

Let $V = \mathbb{R}^3$ and consider the following subsets of V:

$$U = \text{span}\left\{ \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} \right\}, \quad W = \text{span}\left\{ \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} \right\}.$$

1. Determine whether the vectors in U ∪ W are **linearly independent**.
2. Find a **basis** for U and a basis for W
3. Verify whether V=U⊕W, i.e., check whether V is the **direct sum** of U and W.
4. If V=U⊕W, express the vector

$$\mathbf{v} = \begin{pmatrix} 2 \\ 2 \\ 1 \end{pmatrix}$$

as a **unique sum** v=u+w with u ∈ U and w∈W.